$\{x: -1 < x < -1/\sqrt{3},\ 0 < x < 1/\sqrt{3}\}$ there correspond two distinct initial values of $\alpha = L_{n_0}/L_0$, while for solutions of the second type, to each $\beta \in B_4 = \{x: -1/\sqrt{3} < x < 0,\ 1/\sqrt{3} < x < 1\}$ there also correspond two distinct values of α. In /4/ the initial values for the angles λ which correspond to periodic Poincaré solutions are incorrect.

The stability conditions for the solutions are written as

$$\frac{d^2H_0}{dL_0^2}\frac{\partial^2\langle H_1\rangle}{\partial\lambda_0^2} = 3\frac{\omega_0^2}{J_0}(A_1 - C_1)\beta(1+\alpha)\sqrt{1-\beta^2}\times\ \sqrt{1-\alpha^2}\sin 2\lambda_0 > 0 \tag{4.9}$$

$$\frac{\partial^4\langle H_1\rangle}{\partial\lambda_0^4} = -12\omega_0^2(A_1 - C_1)\beta(1+\alpha)\sqrt{1-\beta^2}\sqrt{1-\alpha^2}\sin 2\lambda_0 \neq 0$$

The second of these conditions is satisfied if $\alpha, \beta \neq 0, \pm 1$, while the first condition is equivalent to

$$(A_1 - C_1)\beta \sin 2\lambda_0 > 0 \tag{4.10}$$

From (4.10) it follows that solutions of the first type are orbitally stable if $A_1 > C_1$ and $0 < \beta < 1$, i.e., $-\pi/2 < \vartheta < \pi/2$, or if $A_1 < C_1$ and $-1 < \beta < 0$, i.e., $\pi/2 < \vartheta < 3\pi/2$. Periodic solutions of the second type are orbitally stable if $A_1 > C_1$ and $-1 < \beta < 0$ or if $A_1 < C_1$ and $0 < \beta < 1$. Hence we see that if periodic solutions of the first type are orbitally stable, then solutions of the second type are unstable, and vice versa.

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ON THE IMPULSIVE MOTION OF A RIGID BODY AFTER IMPACT WITH A ROUGH SURFACE*

V.A. SINIDYN

An absolutely rigid plane body in contact with a plane surface of finite area, at each point of which the friction is locally defined by Coulomb's law, with a constant sliding coefficient of friction, is considered. A more precise model of the motion of a body over a rough surface /1/ is obtained. Differential equations of a plane rigid body (a plate) with a circular contact area are derived. The relation between the sliding velocity of the centre of the base area and the angular velocity of the plate is obtained in special cases. The condition under which the instantaneous centre of the base velocity in the course of impulsive motion coincides identically with the base area centre is derived.

The collision between a rigid and a rough surface has been investigated under conditions of point contact (/2/ etc.)

1. Let us consider the basic assumptions made in /1/ on the interaction between a rigid body with a plane base and a plane rough surface, when the body moves on it.

For absolutely rigid bodies and planes the problem is indeterminate, since contact occurs at an infinite number of points. Hence, a small deformation of the surface proportional to

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the local pressure of the body on it is allowed. The contact surface is assumed plane, and its equation

$$z = \alpha x + \beta y + \gamma \tag{1.1}$$

where the coordinate plane XOY coincides with the undeformed plane on which the body moves, and the OX and OY axes are the principal axes of the ellipse of inertia with respect to the centre of mass of the body base area. We call the base of the body the plane figure obtained by projecting the contact points on the plane of the system of coordinates.

Let us find an expressions for the coefficients in Eq.(1.1) /1/. For an element of the base area $d\sigma$, taking into account the choice of coordinate axes, we have

$$\int x\,d\sigma = 0, \quad \int y\,d\sigma = 0, \quad \int xy\,d\sigma = 0 \tag{1.2}$$

We denote by J_x and J_y the moments of inertia of the base area relative to the OX and OY axes

$$J_x = \int y^2\,d\sigma = \sigma\rho_x^2, \quad J_y = \int x^2\,d\sigma = \sigma\rho_y^2 \tag{1.3}$$

(where ρ_x, ρ_y are the radii of intertia of the base area σ). If the elastic forces during the surface deformation obey Hooke's law with proportionality coefficient $\varkappa$, then

$$\varkappa \int z d\sigma = N \tag{1.4}$$

where N is the projection of the plane reaction on the OZ axis.

By d'Alembert principle the following equalities must be satisfied:

$$\mathbf{P} + \mathbf{I} + \mathbf{N} + \mathbf{F} = 0, \quad \mathbf{M}_0{}^p + \mathbf{M}_0{}^i + \mathbf{M}_0{}^n + \mathbf{M}_0{}^f = 0 \tag{1.5}$$

where $\mathbf{P}, \mathbf{I}, \mathbf{M}_0{}^p, \mathbf{M}_0{}^i$ are the principal vectors and principal moments of the active forces and the forces of inertia, and $\mathbf{N}, \mathbf{F}, \mathbf{M}_0{}^n, \mathbf{M}_0{}^f$ are the principal vectors and principal moments of the normal components of reaction and friction forces of the plane.

Suppose that of the active forces only the gravity force acts, and the plane on which the body moves is horizontal. Then, denoting by x_0 and y_0 the coordinates of the projection of the centre of mass of the body on the XOY plane, from (1.5) we obtain

$$\varkappa \int zx\,d\sigma = Px_0 + M_y{}^i, \quad \varkappa \int zy\,d\sigma = Py_0 - M_x{}^i \tag{1.6}$$

From here, using equalities (1.2)–(1.4), we obtain

$$\alpha = \frac{Px_0 + M_y{}^i}{\varkappa\sigma\rho_y{}^2}, \quad \beta = \frac{Py_0 - M_x{}^i}{\varkappa\sigma\rho_x{}^2}, \quad \gamma = \frac{P}{\varkappa\sigma} \tag{1.7}$$

Substituting (1.7) into (1.1) we obtain the equation of contact plane

$$\varkappa z = \frac{P}{\sigma}\left(\frac{x_*}{\rho_y{}^2}x + \frac{y_*}{\rho_x{}^2}y + 1\right) \tag{1.8}$$

$$\left(x_* = x_0 + \frac{M_y{}^i}{P}, \quad y_* = y_0 - \frac{M_x{}^i}{P}\right)$$

When $x_* = y_* = 0$ the pressure is uniformly distributed over the base area. Thus the following statement has been proved: if the moment of the gravity force relative to the centre of mass of the base area is balanced by the projection of the moment of inertia forces on the plane, the pressure is the same at all points of the base. From this follows the theorem in /1/ on the uniform pressure distribution in the case of quiescence. However, the author's indication in /1/ on the applicability of the theorem in the case of motion is not justified.

Then, following the arguments used in /1/, we conclude that the straight lines of equal pressure are parallel to the diameter of the ellipse of inertia, conjugate to the diameter passing through the point (x_*, y_*).

We introduce the system of coordinates $\xi o\eta$ the $o\xi$ axis of which is parallel to the lines of equal pressure, and the $o\eta$ axis of which is perpendicular to them, and polar coordinates r, λ with origin Q at the instantaneous centre of the base velocities (Fig.1).

If we assume that Coulomb's law of dry friction is locally valid for all points of the contact area, then integrating the expression for the elementary friction force

$$dF = \frac{fP}{\sigma}\left(\frac{x_* x}{\rho_y{}^2} + \frac{y_* y}{\rho_y{}^2} + 1\right) d\sigma$$

where f is the coefficient of friction, we have the following expressions /1/ for the projections of the principal vector and principal moment of the friction forces with respect to the centre Q:

$$F_\xi = \iint (H_1\eta + H_2)\frac{\partial r}{\partial\eta}\,d\xi\,d\eta, \quad F_\eta = -\iint (H_1\eta + H_2)\frac{\partial r}{\partial\xi}\,\partial\xi\,d\eta \tag{1.9}$$

$$M_Q = \iint (H_1\eta + H_2) r \, d\xi \, d\eta$$
$$\left(H_1 = \frac{fP}{\sigma}\left(\frac{x_*^2}{\rho_y^4} + \frac{y_*^2}{\rho_x^4}\right)^{1/2}, \quad H_2 = \frac{fP}{\sigma}\right)$$

Such a detailed consideration of the problem of the centre of pressure $C(x_*, y_*)$ is due to the importance of determining the position of this point in the problem of the motion of a rigid body with finite contact area over a surface with friction.

In the special case, when the body is a flat plate, its centre of pressure is at the centre of mass. If in addition $x_* = y_* = 0$ (for instance in the case of a homogeneous plate), the centre of pressure is at the base area centre of mass. In that case expressions (1.9) are substantially simplified, since $H_1 = 0$, and it becomes possible to determine the condition

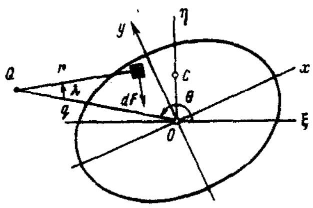

Fig. 1

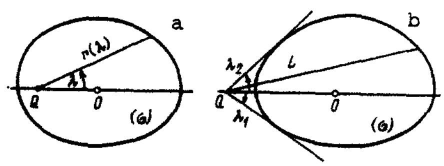

Fig. 2

under which the principal vector of the friction force is directed opposite to the base centre velocity.

For this we will calculate the projection of $\mathbf{F}$ on the radial direction of the polar coordinate system

$$F_r = -\sin\theta \int H_2 \frac{\partial r}{\partial \xi} d\sigma + \cos\theta \int H_2 \frac{\partial r}{\partial \eta} d\sigma \tag{1.10}$$

Substituting the expressions

$$\partial r/\partial\xi = -\cos(\lambda + \theta), \quad \partial r/\partial\eta = -\sin(\lambda + \theta)$$

into (1.10) we obtain

$$F_r = -H_2 \int \sin\lambda \, d\sigma \tag{1.11}$$

Using the representation of an elementary area in polar coordinates $d\sigma = r dr d\lambda$, we change in (1.11) from integration over the area to integration over the contour. As the result, the condition under which the friction force is opposite to the velocity of the centre is obtained in the following form: if the instantaneous centre of the base velocities appertains to the base area (Fig. 2,a) then

$$\int_0^{2\pi} r^2(\lambda) \sin\lambda \, d\lambda = 0 \tag{1.12}$$

but if, however, the point Q lies outside σ (Fig. 2,b), then

$$\int_{\lambda_1}^{\lambda_2} [r_2^2(\lambda) - r_1^2(\lambda)] \sin\lambda \, d\lambda = 0 \quad (r_1(\lambda) = QL, r_2(\lambda) = r(\lambda) - r_1(\lambda)) \tag{1.13}$$

For a non-convex contour and, also, where there is multiple connectedness, the generalizations of (1.12) and (1.13) are obvious.

2. Let us consider the impulsive motion of a plane rigid body when it collides with a plane surface with friction. In addition to the assumptions made in Sect. 1, we assume that the deformation of the surface remains very small under the impact forces (this is achieved by an unlimited increase in the coefficient $\varkappa$ in (1.4)). We further assume the body to be a disk whose base is a circle of radius a. The initial state of the disk at the instant directly preceding the impact is as follows: the disk plane is parallel to the surface plane with which the collision occurs; the disk velocity field has a helical axis perpendicular to the disk plane.

Let us construct the equations of motion of the disk centre of mass and the equation of motion relative to the Koenig axes in their projections on the axes of a cylindrical system of coordinates q, θ, z (Fig. 1) with its centre at the base centre O. The radial direction of the polar coordinates q and θ are determined by the direction on the instantaneous centre of velocities of the base Q. The velocity $\mathbf{V}_c$ of the centre of mass of the disk (point C) is related to the velocity $\mathbf{V}_0$ of the base area centre (point O) by the kinematic relation

$$\mathbf{V}_c = \mathbf{V}_0 + \boldsymbol{\omega} \times \mathbf{OC} \tag{2.1}$$

where ω is the disk angular velocity vector.

In the course of impulsive motion the change in time is negligibly small; hence we select as the independent variable the momentum of the normal component of the reaction of the plane, which we denote by S, with $dS = Ndt$. Taking into account that during the impact the change in the position of the disk can be neglected, and that the coordinate system rotates, we obtain

$$m\left(V\frac{d\theta}{dS} - c\frac{d\omega}{dS}\cos\theta\right) = \Phi_q, \quad m\left(-\frac{dV}{dS} + c\frac{d\omega}{dS}\sin\theta\right) = \Phi_\theta \tag{2.2}$$
$$m\frac{du}{dS} = 1, \quad J_c\frac{d\omega}{dS} = M_c \quad (J_c = m\rho^2)$$

where the following notation is used: m, J_c are, respectively, the disk mass and moment of inertia about the CZ axis (ρ is the radius of gyration), c is the distance of point O from the centre of mass, $-V$ is the projection of the velocity of point O on the transverse direction, u, ω are, respectively, the projections of the velocity of the centre of mass and of the disk angular velocity on the OZ axis, and Φ_q, Φ_θ, M_c are the projections of the principal vector and the principal moment of the friction forces referred to the normal reaction N.

The particular choice of the initial state and plane shape of the body ensure that the projections of the angular velocity of the body on the other two axes are identically zero, and hence their equations are omitted. The third of Eqs.(2.2) is readily integrable and shows that the momentum S, taken as the independent variable, is proportional to the increment of u. Hence the limits of variation of S are determined by the change in the velocity u from some initial state (preimpact), which must be negative $u^- < 0$, to zero (the first phase of impact /2/). The second phase is determined by the elastic properties of the interaction between the disk material and surface, and can be defined by the coefficient of restitution $0 \leqslant \varepsilon \leqslant 1$, which enables us to determine the post-impact velocity $u^+ = -\varepsilon u^-$ and, consequently, the instant when impulsive motion ceases.

Let us consider the special case when the centre of mass of the disk coincides with the base centre $(c = 0)$. Then $\Phi_q = 0$, and Φ_θ and M_c follow directly from formulas in /3/. From Eqs.(2.2) we obtain $\theta = \text{const}$ and the equations

$$\frac{dV}{d\Omega} = \mu\frac{kf_1(k)}{f_3(k)}, \quad k = \frac{V}{\Omega}, \quad V \leqslant \Omega \tag{2.3}$$
$$\frac{dV}{d\Omega} = \mu\frac{f_1(k_1)}{f_2(k_1)}, \quad k_1 = \frac{\Omega}{V}, \quad \Omega \leqslant V$$
$$\Omega = \omega a, \quad \mu = 3\rho^2/a^2$$
$$f_1(k) = (k^{-2} + 1)E(k) - (k^{-2} - 1)K(k)$$
$$f_2(k) = k^{-1}[(4 - 2k^{-2})E(k) - (k^{-2} - 1)(3k^2 - 2)K(k)]$$
$$f_3(k) = (4 - 2k^2)E(k) - (1 - k^2)K(k)$$

where $K(k)$ and $E(k)$ are the complete Legendre elliptic integrals of the first and second kind with modulus $0 \leqslant k \leqslant 1$.

The behaviour of the function f_1, f_2, f_3 was considered in /4/ when investigating the admissibility of neglecting the finite dimensions of the contact area of a rolling body and a plane. It was shown that in the interval $0 \leqslant k \leqslant 1$ these functions are monotonic and of constant sign. Hence, it is possible to limit the consideration to the integral curves (2.3) only in one quadrant of the plane $V\Omega$. The straight line $V = \Omega$ divides the quadrant into two regions in each of which all integral curves can be derived from one integral curve, using the similitude transformation, with the centre of similitude at the origin of coordinates. The coordinate axes $V = 0, \Omega = 0$ are the trajectories of the imaging point. Hence all trajectories of that point pass through the origin of coordinates. Consequently, we come to to conclusion that when the initial centre velocity (the centre of mass is at the centre of the circular base), and the angular velocity are non-zero, during impact they can only vanish simultaneously and remain equal to zero.

We will linearize the right-hand sides of Eqs.(2.3) for small k, taking into account the expansion in series of the elliptic integrals in powers of k^2 in the neighbourhood of the point $k = 0$. We obtain the following approximate equations and respective relations between V and ω for the two cases:

a) $V \ll a\omega \quad (\omega \geqslant 0)$ (2.4)

$$mV' = -fV/\Omega, \quad J_c\omega' = -2af/3, \quad mu' = 1; \quad V = V^-(\omega/\omega^-)^{\mu/2}$$

b) $V \gg a\omega \quad (\omega > 0)$

$$mV' = -f, \quad J_c\omega' = -\frac{5}{6}fa\frac{\Omega}{V}, \quad mu' = 1; \quad V = V^-(\omega/\omega^-)^{2\mu/5} \tag{2.5}$$

where the prime denotes derivatives with respect to S.

It is interesting to compare (2.4) and (2.5) with the hypotheses of the impact of so-called plane particles (non-rotating) with a surface with friction.

In case a) the angular velocity ω^+ at the instant the impact ends (to be specific we will assume the impact to be absolutely inelastic) is related to the initial velocity of approach $u_0=-u^-$ by the equation

$$\omega^+ = \omega^- - \frac{2af}{3\rho^2} u_0$$

hence, from the last of formulas (2.4) we have

$$V^+ = V^-\left(1 - \frac{2f}{\mu}\frac{u_0}{a\omega^-}\right)^{\mu/2}$$

For plane bodies, without slippage or partial slippage, in technical calculations the hypothesis $x_+^{\cdot} = x_-^{\cdot}(1-\lambda)$, where $x_-^{\cdot}, x_+^{\cdot}$ are the relative velocities before and after collision, and λ is an empirical coefficient, is recommended in /5/.

In case b) from Eqs.(2.5) we obtain

$$V^+ - V^- = -f(u^+ - u^-)$$

This case corresponds to impact with complete slippage for which the hypothesis /5/ $x_+^{\cdot} - x_-^{\cdot} = f'(y_-^{\cdot} - y_+^{\cdot})$ is applied, where f' is a coefficient obtained experimentally, and $y_-^{\cdot}, y_+^{\cdot}$ are the components of the particle velocity normal to the surface before and after the impact. The conclusions that can be drawn from the comparison are obvious.

When $c \neq 0$, the expressions on the right-hand sides of Φ_q, Φ_θ, M_c are obtained using the results from /1/, taking into account the remark in Sect. 1

$$\Phi_q = h_1(\varphi_2 - \varphi_3)\cos\theta, \quad \Phi_\theta = -h_1(\varphi_2 + \varphi_3 - q\varphi_1)\sin\theta + h_2\varphi_1^{\cdot} \tag{2.6}$$

$$M_c = -h_1(-\varphi_4 + 2q\varphi_2)\sin\theta - 2h_2\varphi_2 + \Phi_q c\cos\theta + \Phi_\theta(q - c\sin\theta); \quad h_1 = \frac{4fc}{\pi a^4}, \quad h_2 = \frac{f}{\pi a^2}$$

(The functions $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ are denoted in Sect.1 by f_1, f_2, f_3, f_4).

When $c \neq 0$ we obtain for motions for which $\theta = \pi/2$ from Eqs.(2.2), using (2.6), the homogeneous differential equation

$$-\frac{m}{J_c}\frac{dV}{d\omega} + \frac{mc}{J_c} = \frac{\Phi_\theta}{M_c} \tag{2.7}$$

where Φ_θ and M_c are calculated for $\theta = \pi/2$.

For small V/Ω we obtain from (2.7) by linearization the approximate equation

$$dV/d\Omega = \delta + \nu V/\Omega \tag{2.8}$$

whose solution has the form

$$V = C\Omega^\nu - \frac{\delta}{\nu - 1}\Omega \tag{2.9}$$

where C is the constant of integration, and the constant coefficients δ and ν are obtained using the coefficients of expansion of the functions $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ in series

$$\varphi_1 = a_1 q + \ldots, \quad \varphi_2 = a_0 + a_2 q + \ldots, \varphi_3 = a_3 q + \ldots, \varphi_4 = a_4 q + \ldots;$$

$$a_1 = \pi a, \quad a_0 = \pi a^3/3, \quad a_2 = a_3 = 0, \quad a_4 = \pi a^3, \quad q = V/|\omega|$$

$$\delta = \zeta\left(\frac{2\vartheta^2}{2\zeta^2 - 1} + 1\right), \quad \nu = \frac{13\vartheta^2}{2(2\zeta^2 - 1)^2}, \qquad \zeta = c/a, \quad \vartheta = \rho/a$$

When $\theta = -\pi/2$ $(V \ll \Omega)$, we obtain the dependence of V on Ω by substituting $\delta \to -\delta$ into (2.8) and (2.9).

Since the contact over the whole area of the circle is only possible when $c \leqslant a/4$ (see (1.8), we have $2\zeta^2 - 1 \neq 0$.

Note that the results of the function f_4 in /1,p.151/ do not correspond to the expression for that function in /1, p.150/.

The conditions $\theta = \pm\pi/2, V \ll \Omega$, under which solution (2.9) was obtained mean that in the course of impulsive motion the instantaneous centre of the base velocities is a line perpendicular to the lines of equal pressure near the centre of the circle (point O). The non-coincidence of the centre of pressure and the base centre leads to the conclusion that, generally, the base centre velocity may vanish not only when $\omega = 0$, as was the case when $c = 0$.

Let us establish one more property of impulsive motion $(c \neq 0)$, namely that the instantaneous centre of the base velocities coincides identically with the centre of the circular base in the course of impulsive motion $(V \equiv 0)$, if the condition

$$2(\rho^2 + c^2) = a^2 \tag{2.10}$$

is satisfied. Relation (2.10) can be obtained using the equations of rotation of the disk under the action of impact forces due to friction

$$J_0\Delta\omega = -MS;\quad J_0 = J_c + mc^2 \tag{2.11}$$

where $\Delta\omega$ is the angular velocity increment during the impact.

The equation of motion of the centre of mass leads in this case to the equation

$$mc\Delta\omega = -\Phi_\xi S \tag{2.12}$$

Under conditions of the motion considered here M and Φ_ξ should be taken in (2.11) and (2.12) in the form /1/.

$$M = 2h_2a_0,\quad \Phi_\xi = h_1a_0$$

From (2.11) and (2.12) we obtain the linear dependence of the angular velocity increment on the normal reaction momentum

$$\Delta\omega = -\frac{4f}{3am}S = -\frac{4f}{3a}\Delta u$$

where in addition to (2.10) the value of the coefficient $a_0 = \pi a^3/3$ is taken onto account.

For practical purposes we will formulate this property in the form of a statement. If a plane rigid body rotates about an axis passing through the point O normal to the body plane, then at the instant of collisional start of frictional braking (along the axis of rotation) with circular contact area, the axis does not experience transverse impact loads when $a^2 = 2lc$, where l is the reduced length of the physical pendulum.

The last equation follows from a comparison of (2.10) with the requirement that the momentum of the resultant friction forces is applied at the centre of impact. If, for example, $c = a/4$ (the line of zero pressure touches the contour of the circular contact area), the diameter of that circle must be equal to the reduced length. In the trivial case when $c = 0$ we have $V \equiv 0$ for any value of a.

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RELAXATION IN DISSIPATIVE MECHANICAL SYSTEMS*

L.D. ESKIN

An asymptotic expression for long times is obtained for a $2n$-parametric family of solutions of a Hamiltonian system with n degrees of freedom, modified by the addition of generalized dissipative forces. The method used here is based on a preliminary study of the solutions of a linearized system of equations, followed by the application of the Schauder principle in Banach space with a suitably chosen norm.

1. The aim of this paper is to study relaxation in a mechanical system, the equations of motion of which are written in the form

$$p^{\cdot} = -\frac{\partial H(p,q)}{\partial q} + Q(t,p,q),\quad q^{\cdot} = \frac{\partial H(p,q)}{\partial p} \tag{1.1}$$
$$H(p,q) = 2^{-1}(p^2 + q^2) + \Pi_1(q)$$

Here p, q are $(n \times 1)$-vectors (columns) of generalized momenta and coordinates, and the $(n \times 1)$-vector $Q(t, p, q)$ defines the Lagrangian forces in $t \in R_+, p, q$ variables. The expansion $\Pi_1(q)$ in q_i coordinates begins with terms of at least the third order.

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